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CAUSALLY CONSTRAINED ACTIVE SOUND POWER CONTROL IN AN ENCLOSED SPACE

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In this paper the causality problem is considered of an active sound power control system that attempts to reduce total acoustic potential energy in an enclosed space by total source power minimization. To find the causally constrained optima under the white random sound field, time domain analysis based on the classical Wiener filtering technique for the active sound power control system in the enclosed sound field is accomplished. For simplicity, a two-monopole source system surrounded by an arbitrary acoustic impedance condition at the enclosure boundary is considered, which turns out to be a multiple-error Wiener filtering problem that has six errors. Theoretical causal and non-causal optimal filters for the white random noise are derived, and their frequency characteristics are also analyzed by the spectral factorization theorem. Simple numerical simulation for a one-dimensional sound field considering plane waves is performed to compare the control performance of non-causal filtering with that of causal filtering. The simulation result shows an important characteristic of the causally constrained optimal filtering; the basic acoustic controllability determined by the wavelength and location of control source is still maintained, even if the control performances in the overall frequency range are degraded due to the lack of predictability of the primary random excitation signal. The simulation result also reveals that large portions of the control source power show negative power, which means that the control source behaves as an active sound power absorber. This result is different from the well-known property of an optimal control source to minimize the total source power output in the deterministic sound field where the primary excitation is harmonic; i.e., the control source neither radiates nor absorbs any net acoustic power under optimal conditions to minimize the total sound power of the sources.

1. INTRODUCTION

Various techniques for active control of sound fields have received considerable attention recently. With regard to the active control of sound in *enclosed fields*, the prevalent theoretical basis has relied mostly on the minimization of the *acoustic potential energy* present in the enclosure [1]. In parallel with the acoustic energy-based active control method, the *sound power* (of sources) based active control strategy has also been developed [2–4]. Most of the research on active sound power control has been directed towards the sound radiation problem in *free fields*.

Although these two strategies of minimizing energy and minimizing source power output use very different criterion to adjust the control sources, it has been demonstrated numerically for one-dimensional sound field that the effects in an enclosure of minimizing

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total acoustic potential energy, and of minimizing total source power output, are similar [3]. Johnson and Elliott [5] have also presented a sound power control method using the theoretical property of an optimal control source; if acoustic reciprocity exists between each point on the primary source and each point on the control source then under optimal conditions to minimize total source power output the acoustic power output of the control source is *exactly zero* [3, 4].

In a recent study by the authors [6], it has been found theoretically as well as experimentally that the source power minimization gives good results for total acoustic potential energy reduction, *if the enclosed field is lightly damped* with a frequency range which includes the low modal density as well as high modal density range. (In general, it is not always true in every sound field which has arbitrary boundary conditions, because the sound power of sources and sound energy in a surrounding volume are related by the acoustic energy balance). Thus, the active sound power control can provide an alternative practical means for global noise reduction in an enclosed space. It can also augment the many useful results for active sound power control in a free field which have been dealt with in most previous studies.

In a previous work [6], an acoustic field analysis in the frequency domain assuming deterministic excitation, such as single or multiple sinusoidal excitation, was employed to assess the *acoustic possibility* of global noise reduction (or total acoustic potential energy reduction) in enclosures by sound power control, and to investigate the *basic characteristics* of the active sound power control method. The frequency domain optimization method would be adequate for predicting the performance of an active control system, if one were to deal with deterministic sound fields. However, this method cannot necessarily be applied in cases in which the primary sound field is stationary random in nature. This is because the frequency domain optimization often yields a physically unrealizable optimal control strategy that requires the control sources to act non-causally with respect to the primary sources. That is, the secondary sources involved may have to emit signals prior to the emission of signals by the primary source if optimal results are to be achieved. For example, the active minimizations of total sound power due to a primary monopole source and a secondary monopole source in a free field [2] or a one-dimensional infinite duct field [7] require non-causal optimal filters, as pointed out in references [2, 7]. In other words, the system causality is not a constraint when the primary excitation is sinusoidal, because future values of the primary excitation signal are completely predictable under steady state conditions. However, the causality becomes an important issue in random sound field control, because future values of the excitation cannot be predicted.

On the causality in the active sound and vibration control system, several analytic studies have been reported in the literature [8–10]. Nelson *et al*. [8] have developed a time-domain formulation for the case of active control of sound, and dealt with the problem of minimizing the total acoustic potential energy in an enclosure and the minimization problem of two point source power output in a free field. Based on Nelson *et al*.'s formulation, Joplin *et al*. [9] accomplished a numerical calculation to find the optimum causal filter for the active control of low frequency random sound in enclosures. Burdisso *et al*. [10] carried out a causality analysis of feedforward point-vibration control of elastic systems by frequency domain formulation. However, there has not been any attempt to evaluate the degree of the total acoustic potential energy reduction in enclosed fields when subjected to random excitation by applying *causally constrained sound power control*.

In this paper the active global control is considered of random sound field in an enclosure generated by a stationary random noise source. The control approach used is focused on the minimization of total source power output (not on the direct minimization of total acoustic potential energy in the control volume). Its effectiveness is assessed in terms of acoustic potential energy that remains in the enclosure when the control source is operating. The main purpose of this study is to determine the causally constrained optimal characteristics of the active sound power control system for reducing the total acoustic potential energy in an enclosed space. The theoretical approach adopted is the time domain formulation method, based on the classical Wiener filtering technique [11, 12]. The frequency characteristics of the causally constrained optimal filter are also derived to compare them with the characteristics of the unconstrained optimal filter. A numerical example for a simple one-dimensional lightly damped random sound field is presented to compare the control results of the causal filter with those of the unconstrained filter.

2. WIENER FILTERING ANALYSIS OF ACTIVE SOUND POWER CONTROL

2.1. A TWO-SOURCE POWER CONTROL SYSTEM IN ENCLOSED SPACE

We consider a single channel source power control system in an enclosed sound field with arbitrary acoustic impedance condition at the enclosure walls ($Z_W(\omega)$), as illustrated in Figure 1. The primary source strength (or volume velocity) fluctuation $q_p(t)$ is a stationary random signal and $q_s(t)$ is a coherent secondary source strength fluctuation. Hereafter, subscripts p and s indicate primary source and secondary control source respectively. For a systematic analysis that includes the arbitrary shape of the enclosure and the arbitrary boundary condition of the enclosure walls, the impulse response function of the system is expressed by an acoustic Green function in the time domain, which is a unit solution of the acoustic wave equation. Green function $g_{ij}(t)$ denotes the acoustic pressure response at $\vec{r} = \vec{r}_i$ due to a source of unit strength impulse applied at $\vec{r} = \vec{r}_j$, and it satisfies the acoustic impedance boundary condition of the enclosure walls. The general multi-source situation would be a simple extension of this case as long as one handles linear acoustics.

In this case, we now wish to find the optimal control filter $h_0(t)$ to minimize the total acoustic power output of the sources. We also wish to calculate the best that can be achieved in reducing the total acoustic potential energy in the enclosure when the optimal filter is operating.

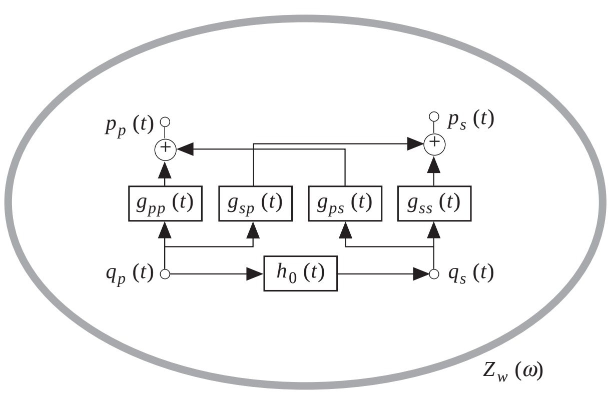

Figure 1. The active sound power control problem in the time domain with an arbitrary acoustic impedance condition at the enclosure walls ($Z_w(\omega)$), composed of a primary source, the strength of which is $q_p(t)$, and a secondary source $q_s(t)$. The $g_{ij}(t)$ is the acoustic Green function in the time domain, which means the acoustic pressure response at $\vec{r} = \vec{r}_i$ due to a source of unit strength impulse applied at $\vec{r} = \vec{r}_j$, and $h_0(t)$ represents the active controller; an optimal filter relates the control source output to that of the primary source.

The filter output, that is, the secondary source strength, can be expressed as

$$q_s(t) = h_0(t) * q_p(t), \tag{1}$$

where the asterisk represents the convolution process. The acoustic pressure fluctuations on the primary source and secondary source, $p_p(t)$ and $p_s(t)$, can be expressed as

$$p_p(t) = g_{pp}(t) * q_p(t) + g_{ps}(t) * q_s(t), \qquad p_s(t) = g_{sp}(t) * q_p(t) + g_{ss}(t) * q_s(t). \tag{2a, b}$$

The cost function to be minimized is the total acoustic radiation power of the sources. Assuming a stationary random process, the time averaged total acoustic power due to the primary and secondary sources can be expressed as

$$J = \mathrm{E}[q_p(t) \quad p_p(t)] + \mathrm{E}[q_s(t) \quad p_s(t)], \tag{3}$$

where $\mathrm{E}[\cdot]$ denotes the ensemble average operator. In equation (3), the first term represents the power output of the primary source and the second term the power output of the secondary source.

2.2. WIENER–HOPF INTEGRAL EQUATION FOR THE RANDOM SOUND FIELD

In order to formulate the present minimization problem (the cost function of which is described by equation (3)) into a Wiener filtering one, one can use the following identity

$$\mathrm{E}[f(t)g(t)] = \tfrac{1}{2}\mathrm{E}[f^2(t) + g^2(t) - \{f(t) - g(t)\}^2]. \tag{4}$$

Then the cost function can be rewritten as

$$J = \tfrac{1}{2}\mathrm{E}[q_p^2(t) + p_p^2(t) - \{q_p(t) - p_p(t)\}^2] + \tfrac{1}{2}\mathrm{E}[q_s^2(t) + p_s^2(t) - \{q_s(t) - p_s(t)\}^2]. \tag{5}$$

Substitution of equations (2a) and (2b) into equation (5), and appropriate algebraic manipulation gives

$$J = \mathrm{E}[p_{pp}(t)\, q_p(t)] + \tfrac{1}{2}\mathrm{E}[q_p^2(t)] + \tfrac{1}{2}\mathrm{E}[p_{sp}^2(t)] + \mathrm{E}[q_s^2(t)] + \tfrac{1}{2}\mathrm{E}[p_{ps}^2(t)] + \tfrac{1}{2}\mathrm{E}[p_{ss}^2(t)]$$
$$- \tfrac{1}{2}\mathrm{E}[\{p_{ps}(t) - q_p(t)\}^2] - \tfrac{1}{2}\mathrm{E}[\{p_{sp}(t) - q_s(t)\}^2] - \tfrac{1}{2}\mathrm{E}[\{p_{ss}(t) - q_s(t)\}^2], \tag{6}$$

where

$$p_{ij} = g_{ij}(t) * q_j(t), \qquad i, j = p \text{ or } s. \tag{7}$$

We now wish to find the optimal filter that minimizes the total acoustic power output, which is expanded in nine terms as in equation (6). The first three terms in equation (6) are not a function of the optimal filter $h_0(t)$. That is, these terms can be considered as constant, and not affected by the optimal filter. On the other hand, the remaining six terms are affected by active control filter $h_0(t)$; that is, they are a function of the optimal filter. Therefore, the cost function (that is, the total source power output) consists of a weighted sum of six mean-squared errors and a constant,

$$J = \sum_{i=1}^{6} w_i \mathrm{E}[e_i^2(t)] + C, \tag{8}$$

where the w_i are weighting factors, the e_i are error signals, and C is a constant:

$$w_1 = 1, \qquad e_1(t) = -q_s(t) = -q_p(t) * h_0(t), \tag{9a, b}$$

$$w_2 = \tfrac{1}{2}, \qquad e_2(t) = -p_{ps}(t) = -q_p(t) * h_0(t) * g_{ps}(t), \tag{9c, d}$$

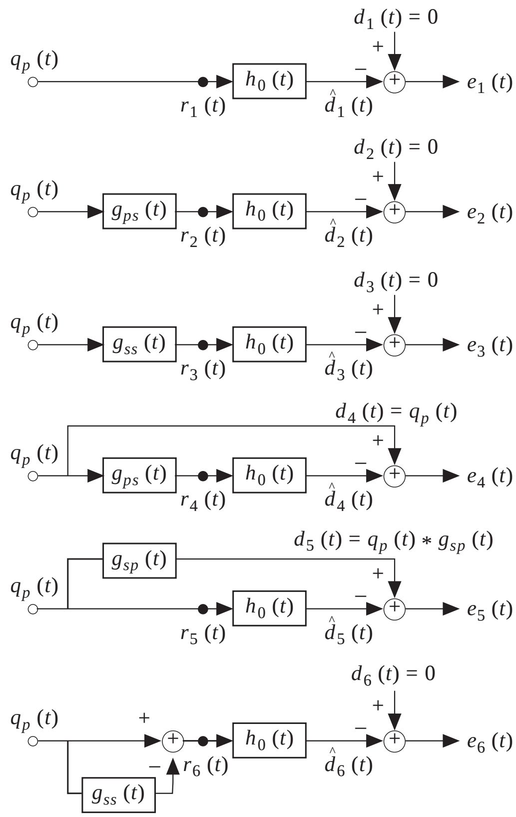

Figure 2. Interpretation of the active sound power control problem in terms of the six-error Wiener filtering problem. One wishes to determine the optimal filter $h_0(t)$ that minimizes the weighted sum of squared errors (see equations (8) and (9)): $J = \mathrm{E}[e_1^2(t)] + \frac{1}{2}\mathrm{E}[e_2^2(t)] + \frac{1}{2}\mathrm{E}[e_3^2(t)] - \frac{1}{2}\mathrm{E}[e_4^2(t)] - \frac{1}{2}\mathrm{E}[e_5^2(t)] - \frac{1}{2}\mathrm{E}[e_6^2(t)] + C$.

$$w_3 = \tfrac{1}{2}, \qquad e_3(t) = -p_{ss}(t) = -q_p(t) * h_0(t) * g_{ss}(t), \tag{9e, f}$$

$$w_4 = -\tfrac{1}{2}, \qquad e_4(t) = q_p(t) - p_{ps}(t) = q_p(t) - q_p(t) * h_0(t) * g_{ps}(t), \tag{9g, h}$$

$$w_5 = -\tfrac{1}{2}, \qquad e_5(t) = p_{sp}(t) - q_s(t) = q_p(t) * g_{sp}(t) - q_p(t) * h_0(t), \tag{9i, j}$$

$$w_6 = -\tfrac{1}{2}, \qquad e_6(t) = p_{ss}(t) - q_s(t) = q_p(t) * h_0(t) * g_{ss}(t) - q_p(t) * h_0(t), \tag{9k, l}$$

$$C = \mathrm{E}[p_{pp}(t) \quad q_p(t)] + \tfrac{1}{2}\mathrm{E}[q_p^2(t)] + \tfrac{1}{2}\mathrm{E}[p_{sp}^2(t)]. \tag{9m}$$

Assuming a time invariant process, one reconstructs each error signal into the desired signal ($d_i(t)$) which is not influenced by the optimal filter $h_0(t)$, and the estimated signal ($\hat{d}_i(t)$) which is filtered by $h_0(t)$:

$$e_1(t) = d_1(t) - \hat{d}_1(t) = 0 - q_p(t) * h_0(t), \tag{10a}$$

$$e_2(t) = d_2(t) - \hat{d}_2(t) = 0 - q_p(t) * g_{ps}(t) * h_0(t), \tag{10b}$$

$$e_3(t) = d_3(t) - \hat{d}_3(t) = 0 - q_p(t) * g_{ss}(t) * h_0(t), \tag{10c}$$

$$e_4(t) = d_4(t) - \hat{d}_4(t) = q_p(t) - q_p(t) * g_{ps}(t) * h_0(t), \tag{10d}$$

$$e_5(t) = d_5(t) - \hat{d}_5(t) = q_p(t) * g_{sp}(t) - q_p(t) * h_0(t), \tag{10e}$$

$$e_6(t) = d_6(t) - \hat{d}_6(t) = 0 - q_p(t) * (1 - g_{ss}(t)) * h_0(t). \tag{10f}$$

Hence, one can now interpret the problem in terms of the multiple-error Wiener filtering problem which has six errors and a constant, as illustrated in Figure 2. In this Wiener filtering problem, the received reference signals ($r_i(t)$, $i = 1, 2, \ldots, 6$) can be expressed as

$$r_1(t) = r_5(t) = q_p(t), \qquad r_2(t) = r_4(t) = q_p(t) * g_{ps}(t), \tag{11a, b}$$

$$r_3(t) = q_p(t) * g_{ss}(t), \qquad r_6(t) = q_p(t) * (1 - g_{ss}(t)). \tag{11c, d}$$

By applying the weighted multiple-error Wiener filtering technique outlined in reference [8],

$$\sum_{i=1}^{6} w_i R_{rdi}(t) - \int_{t_0}^{T} h_0(\tau) \sum_{i=1}^{6} w_i R_{rri}(t-\tau)\, \mathrm{d}\tau = 0, \tag{12}$$

where the correlation functions are $R_{rdi}(\tau) = \mathrm{E}[r_i(t-\tau)d_i(t)]$ and $R_{rri}(\tau) = \mathrm{E}[r_i(t-\tau)r_i(t)]$, one can obtain a Wiener–Hopf integral equation that the impulse response of the optimal filter $h_0(t)$ should satisfy:

$$\{g_{ps}(-t) + g_{sp}(t)\} * R_{qpqp}(t) + \int_{t_0}^{T} h_0(\tau)[\{g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)\} * R_{qpqp}(t-\tau)\, \mathrm{d}\tau = 0. \tag{13}$$

Now, we assume that the primary excitation is white random noise with unit spectral density, that is, the autocorrelation function of $q_p(t)$, $R_{qpqp}(\tau) = \delta(\tau)$, to check the lower bound of the active control performance by a *causally* operating filter. Then, the Wiener–Hopf integral equation becomes

$$g_{ps}(-t) + g_{sp}(t) + \int_{t_0}^{T} h_0(\tau)[g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)]\, \mathrm{d}\tau = 0. \tag{14}$$

Depending on the available data history, Wiener filters can be classified into three types: (1) $t_0 = -\infty$, $T = \infty$, unconstrained IIR (infinite impulse response) Wiener filter; (2) $t_0 = 0$, $T = \infty$, causal IIR Wiener filter; (3) $t_0 = 0$, $T = T_f$, causal FIR (finite impulse response) Wiener filter. In the next three sections, the optimal filters of the above three types for minimizing the total sound power will be derived respectively.

2.3. UNCONSTRAINED IIR FILTERING

From equation (14), the Wiener–Hopf integral equation for the unconstrained (on the causality) optimal filtering can be written as

$$g_{ps}(-t) + g_{sp}(t) + \int_{-\infty}^{\infty} h_0(t)[g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)]\, \mathrm{d}\tau = 0. \tag{15}$$

The characteristics of the optimal filter in the frequency domain can easily be obtained by taking Fourier transform of equation (15); that is,

$$H_0(\omega) = -\frac{G_{ps}^*(\omega) + G_{sp}(\omega)}{G_{ss}^*(\omega) + G_{ss}(\omega)} = -\frac{G_{ps}^*(\omega) + G_{sp}(\omega)}{2R_{ss}(\omega)}, \tag{16}$$

where G_{ps}, and G_{sp} are acoustic transfer impedance functions between primary source and secondary source, G_{ss} is the acoustic radiation impedance of the secondary source, and R_{ss} is the real part of G_{ss}; $R_{ss}(\omega) = \mathrm{Re}\{G_{ss}(\omega)\}$.

If the acoustic reciprocity relation between the two sources holds, that is, $G_{ps}(\omega) = G_{sp}(\omega)$, the transfer function of the unconstrained optimal filter becomes

$$H_0(\omega) = -\frac{R_{sp}(\omega)}{R_{ss}(\omega)}, \tag{17}$$

where $R_{sp}(\omega) = \mathrm{Re}\{G_{sp}(\omega)\}$. In fact, the above result in equation (17) is exactly the same as that obtained by frequency domain analysis in a previous study [3]. In this case, it is also well known that the control source power output under the optimal condition becomes exactly zero [3–5]. That is, it neither radiates nor absorbs any net acoustic power.

However, this unconstrained optimal filter is *not* physically realizable for the active control of the *random* sound field, since it requires future data of the primary source output, which is randomly fluctuating. On the other hand, if a sound field is deterministic, this unconstrained optimal filter is physically realizable, since the prediction of the deterministic signal is possible by the causal delay process.

2.4. CAUSALLY CONSTRAINED IIR FILTERING

From equation (14), the Wiener–Hopf integral equation for *causally* constrained optimal filtering for the white random noise, with unit spectral density, can be written as

$$g_{ps}(-t) + g_{sp}(t) + \int_0^\infty h_0(\tau)[g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)]\,\mathrm{d}\tau = 0, \qquad t \geqslant 0, \tag{18a}$$

$$h_0(t) = 0, \qquad t < 0. \tag{18b}$$

In order to obtain the optimal causal filter from the above integral equation, a well-known spectral factorization process in the textbooks [11–13] can be used. The derivation of the optimal causal solution is presented in full in the Appendix. In this case, however, we cannot directly apply the classical spectral factorization method to solve the integral equation (18a), since the integral equation (18a) includes only impulse response functions. (The standard Wiener–Hopf integral equation consists of correlation functions and an impulse response of optimal filter). Therefore, we need to factorize the acoustic radiation impedance function of the sound source. The optimal causal solution is given by

$$H_0(\omega) = -\left[\frac{1}{2R_{ss}^+(\mathrm{j}\omega)}\right]\left[\frac{G_{ps}(-\mathrm{j}\omega) + G_{sp}(\mathrm{j}\omega)}{R_{ss}^-(\mathrm{j}\omega)}\right]_+, \tag{19}$$

where

$$R_{ss}(\mathrm{j}\omega) = R_{ss}^+(\mathrm{j}\omega)R_{ss}^-(\mathrm{j}\omega), \tag{20}$$

$R_{ss}^{+}(j\omega)$ represents the minimum phase part of $R_{ss}(\mathrm{j}\omega)$ and the remaining $R_{ss}^{-}(\mathrm{j}\omega)$ is the non-minimum phase part, the subscript "+" sign meaning the causal part. If the acoustic reciprocity between two sources holds, then the transfer function for the optimal filter reduces to

$$H_0(\omega) = -\left[\frac{1}{R_{ss}^{+}(\mathrm{j}\omega)}\right]\left[\frac{R_{sp}(\mathrm{j}\omega)}{R_{ss}^{-}(\mathrm{j}\omega)}\right]_{+}. \tag{21}$$

It is noteworthy that these causal filters resulting in equations (19) and (21) are inherently different from the characteristics of the unconstrained optimal filters which were derived by unconstrained filtering in section 2.3 (see equations (16) and (17)) or by frequency domain analysis in a previous study [3].

2.5. CAUSAL FIR FILTERING IN THE DISCRETE TIME DOMAIN

From equation (14), the Wiener–Hopf integral equation for causally constrained optimal FIR filtering can be expressed as

$$g_{ps}(-t) + g_{sp}(t) + \int_0^{T_f} h_0(\tau)[g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)]\,\mathrm{d}\tau = 0, \qquad t \geqslant 0, \tag{22a}$$

$$h_0(t) = 0, \qquad t < 0. \tag{22b}$$

In the discrete time domain, the Wiener–Hopf integral equation can be written as

$$g_{ps}(-n) + g_{sp}(n) + \sum_{i=0}^{L-1} h_0(n)[g_{ss}(-n+i) + g_{ss}(n-i)] = 0, \qquad n \geqslant 0, \tag{23a}$$

$$h_0(n) = 0, \qquad n < 0, \tag{23b}$$

where L is the finite record length.

In equation (23a), the transfer impulse response between two sources, $g_{ps}(n)$ and $g_{sp}(n)$, are physically causal; i.e., there is no response before excitation. Also, these impulse responses in an enclosure are composed of the delayed combinations of the initial excitation; i.e., linear superposition of a direct wave from a source and many reflected waves from surface boundaries. Therefore, it is clear that $g_{ps}(-n) = 0$ for $n \geqslant 0$ in equation (23a). If the two sources are collocated—in fact, this situation is a trivial case in active noise control—$g_{ps}(0)$ has large value, because the first direct wave component is propagating without finite delay of the excitation.

If we let $h_0(n)$ be the transversal filter of which the number of coefficient is L, then equation (23a) can be expressed in vector form:

$$g_{sp}(n) + \mathbf{g}_{ss}^{\mathrm{T}}\mathbf{h}_0 = 0, \qquad n \geqslant 0, \tag{24}$$

where

$$\mathbf{g}_{ss}(n) = \begin{bmatrix} \mathrm{g}_{ss}(-n) + \mathrm{g}_{ss}(n) \\ \mathrm{g}_{ss}(-n+1) + \mathrm{g}_{ss}(n-1) \\ \vdots \\ g_{ss}(-n+L-1) + g_{ss}(n-L+1) \end{bmatrix}, \tag{25a}$$

$$\mathbf{h}_0 = [h_0 h_1 \cdots h_{L-1}]^{\mathrm{T}}, \tag{25b}$$

and, written in matrix form truncated by L,

$$\begin{bmatrix} g_{sp}(0) \\ g_{sp}(1) \\ g_{sp}(2) \\ \vdots \\ g_{sp}(L-1) \end{bmatrix} + \begin{bmatrix} 2g_{ss}(0) & g_{ss}(1) & g_{ss}(2) & \cdots & g_{ss}(L-1) \\ g_{ss}(1) & 2g_{ss}(0) & g_{ss}(1) & \cdots & g_{ss}(L-2) \\ g_{ss}(2) & g_{ss}(1) & 2g_{ss}(0) & \cdots & g_{ss}(L-3) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g_{ss}(L-1) & g_{ss}(L-2) & g_{ss}(L-3) & \cdots & 2g_{ss}(0) \end{bmatrix}$$

$$\times \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_{L-1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{26}$$

We denote the matrix equation ($L \times L$ matrix version of the normal Wiener–Hopf equation) as

$$\mathbf{g}_{sp} + \mathbf{G}_{ss}\mathbf{h}_0 = \mathbf{0}. \tag{27}$$

Then the optimal FIR filter can be obtained as

$$\mathbf{h}_0 = -\mathbf{G}_{ss}^{-1}\mathbf{g}_{sp}. \tag{28}$$

In fact, the elements $g_{ss}(n)$ of matrix $\mathbf{G}_{ss}$ denote the sound pressure responses on the secondary source due to its own excitation. In other words, $g_{ss}(n)$ represents the self-radiation process of the secondary source. Thus, we will call the matrix G_{ss} the "self-radiation matrix", which is analogous to the auto-correlation matrix in signal processing. The self-radiation matrix is symmetric and takes the form of a Toeplitz matrix, the elements of which along the same diagonal have the same value. However, the self-radiation matrix in general is not non-negative definite, which is different from the auto-correlation matrix. The self-radiation matrix has the largest values of the diagonal terms, since $g_{ss}(0)$ in diagonal terms means the first direct wave without any time delay or amplitude decay due to boundary reflections. Exploiting these properties of the self-radiation matrix, the above matrix equation (28) can be solved efficiently using Levinson's algorithm [14].

2.6. PERFORMANCE BOUND

The optimal causal IIR Wiener solution obtained in section 2.4 provides a criterion for the lower bound of the control performance that can be achieved by an active sound power control system. As the primary sound field becomes deterministic, the power control performance will be better, since the sound field is more predictable. When a sound field is subject to purely deterministic excitation such as single or multiple sinusoidal excitation, the upper bound can be determined by the unconstrained IIR Wiener filter as in section 2.3. The causal optimal FIR Wiener filter in the discrete time domain (section 2.5) can provide us with useful information for the active sound power control system before real-time adaptive filtering implementation, because the adaptive filter must converge to the theoretical causal optimal FIR filter for successful adaptive feedforward control.

In general, the practical sound fields to be actively controlled have random characteristics as well as deterministic characteristics. For example, the engine-induced interior noise of road vehicles is composed of harmonic components, which are strongly correlated with the periodic reciprocal motion of the engine and other random noise. Another typical example is fan noise, which has harmonic components of BPF (blade passage frequency) related to the rotational speed and the number of blades; and aeroacoustic flow noise, which is random in nature. Therefore, in general sound fields, the performance of the active power control would be limited by unconstrained optimal filtering (upper bound) and causally constrained optimal filtering (lower bound) depending on the nature of the signal. In fact, in any real implementation, there will be additional factors that may further degrade the control performance: a finite sampling time, the number of filter coefficients, the measurement noise, and so on.

3. SIMULATION

In order to investigate controllability by the *causally constrained* sound power control for the random sound field in an enclosed space, we consider a one-dimensional circular duct which has a primary source and a secondary one, as illustrated in Figure 3. The plane waves are assumed to have wavelengths of interest that are much longer than the dimension of the duct cross-section.

In this duct model, the Green function which represents the acoustic pressure response at $x = x_i$ due to a source of unit strength applied at $x = x_j$, and which satisfies all the boundary conditions; can be readily obtained as

$$G_{ij}(k) = \frac{\rho_0 c}{2S} \frac{\{R_0\,\mathrm{e}^{-\mathrm{j}kx_i} + \mathrm{e}^{\mathrm{j}kx_i}\}\{\mathrm{e}^{\mathrm{j}k(L-x_j)} + R_L\,\mathrm{e}^{-\mathrm{j}k(L-x_j)}\}}{\mathrm{e}^{\mathrm{j}kL} - R_0 R_L\,\mathrm{e}^{-\mathrm{j}kL}}, \qquad 0 \leqslant x_j \leqslant x_i, \tag{29a}$$

$$G_{ij}(k) = \frac{\rho_0 c}{2S} \frac{\{\mathrm{e}^{\mathrm{j}k(L-x_i)} + R_L\,\mathrm{e}^{-\mathrm{j}k(L-x_i)}\}\{R_0\,\mathrm{e}^{-\mathrm{j}kx_j} + \mathrm{e}^{\mathrm{j}kx_j}\}}{\mathrm{e}^{\mathrm{j}kL} - R_0 R_L\,\mathrm{e}^{-\mathrm{j}kL}}, \qquad x_i \leqslant x_j \leqslant L, \tag{29b}$$

where k is the wavenumber and R_0 and R_L are the complex reflection coefficients at the ends.

In this simulation, the lightly damped boundary condition in which the reflection coefficients on both ends are 0·9 was considered. In this instance, we would like to emphasize that the control source location in the simulation configuration is not for the cancellation of downstream noise in the traditional duct noise problem, but rather for investigation of the effect of the control source location on the total sound power control of sources.

For the white random sound field, we calculated the total acoustic potential energy attenuation (which is the ultimate goal of the active sound power control in this paper) as well as the total sound power attenuation itself, by unconstrained optimal filtering (section 2.3) and causally constrained optimal filtering (sections 2.4 and 2.5). Derivation

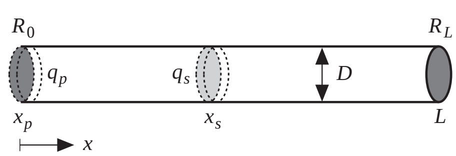

Figure 3. The one-dimensional finite circular duct model in which a primary source is located at x_p with strength q_p and a secondary source at x_s with q_s. R_0 and R_L are the reflection coefficients at both ends: $L = 1$ m, $D = 0{\cdot}16$ m, $x_p = 0$ m, $x_s = 0{\cdot}4$ m, $q_p = 2 \times 10^{-3}$ m^3/s, and $R_0 = R_L = 0{\cdot}9$.

of the theoretical causal filter (section 2.4) was so complicated for the model that we tried to obtain the causally constrained optimal FIR filter (section 2.5) which has a sufficient number of filter coefficients. The speed of sound was taken to be 343 m/s, the sampling frequency was 3·43 kHz, and the number of FIR filter coefficients was taken to be 100, which is large enough to model the decay of the impulse response of the main acoustic plant.

In Figure 4(a) are illustrated the total sound power distribution, before and after unconstrained optimal filtering (section 2.3) and causal optimal FIR filtering (based on the Wiener–Hopf matrix equation (28) in section 2.5) have been applied. In Figure 4(b) are shown the total acoustic potential energy distributions. In Figures 4(c) and 4(d) are shown the insertion losses of the active control system; i.e., the level drops caused by its insertion. In Figure 4(e) is shown the coupling function, which represents the controllability of the control source when the sound field is subject to deterministic

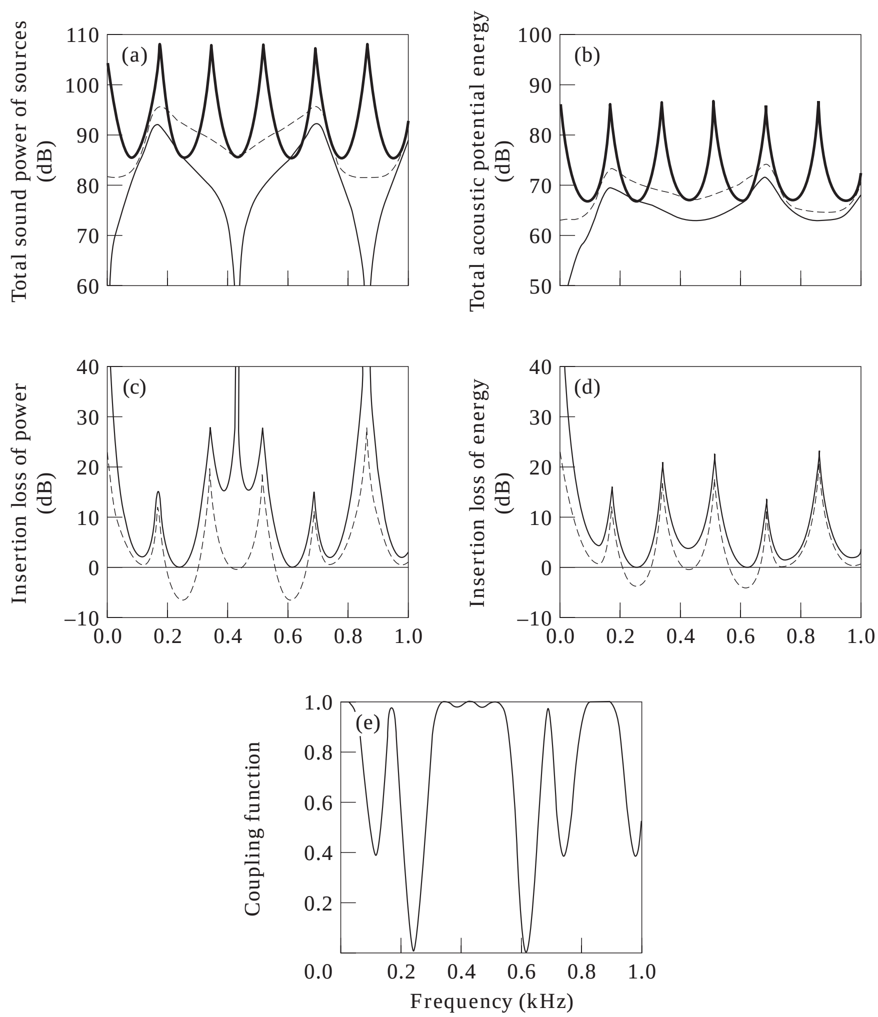

Figure 4. The total sound power(a) and total acoustic potential energy (b) before (——) and after active source sound power control by an unconstrained optimal filter (——) and a causally constrained optimal filter (- - - -), insertion losses due to each filtering (c, d) and the coupling function (e), in the lightly damped one-dimensional duct system (Figure 3).

excitation. The insertion loss (IL) on a decibel scale due to active sound power control is determined by $IL(\omega) = -10\log_{10}\{1 - \eta_{ps}^2(\omega)\}$, where coupling function is expressed as $\eta_{ps}^2(\omega) = R_{ps}^2(\omega)/\{R_{pp}^2(\omega)R_{ss}(\omega)\}$ and $R_{ij}(\omega) = \mathrm{Re}\,(G_{ij}(\omega))\}$ [6].

From the control results by unconstrained filtering (thin solid lines) in Figures 4(a) and 4(b), one can see that the total acoustic potential energy in the enclosed space (Figure 4(b)) as well as the total sound power of the source itself (Figure 4(a)), was reduced by the active sound power control. However, there are some frequency components (230 Hz and 610 Hz) which are not well controlled. We can realize that these components correspond to ones the coupling function values of which are small (Figure 4(e)). In other words, the control source is weakly coupled with the primary excitation field for these frequency components [6]. It is noteworthy that this unconstrained optimal filtering result has two physical meanings. One is that it expresses the control result achieved by a physically realizable causal filter when the primary sound field is deterministic in nature. The other is that it represents the control result due to a non-causal filter (which is in fact unrealizable in practice) when the sound field has a random behavior.

On the other hand, one can see that the causal filtering (dotted lines) gives worse control results than the unconstrained filtering ones (thin solid lines) in Figures 4(a) and 4(b), as expected. However, it is noteworthy that the fundamental controllability, which is determined by the acoustic coupling strength of the control source with the primary source, is still maintained, as can be seen by comparing the insertion loss curves (Figures 4(c) and 4(d)). A more detailed investigation of Figures 4(c) and 4(d) leads to the fact that the coupling function values of all the acoustic resonances, are nearly unities in Figure 4(e); high controllability is greatly suppressed. However, the second and fourth antiresonances (230 Hz and 610 Hz) (coupling function values) are nearly zero in Figure 4(e); low controllability, are rather increased after the control. That is, these frequency components are difficult to control at the control source location. In practical implementation, there might be more degradations of the control performance in these antiresonance frequency components by the weak observability due to the measurement noise.

In Figures 5(a) and 5(b) are represented the frequency characteristics of the unconstrained optimal filter and the causally constrained optimal filter respectively. From Figure 5(a), one can see that the unconstrained optimal filter has only a real part (solid line); that is, the control source is driven exactly in phase or completely out of phase with the primary source. On the other hand, the causally constrained optimal filter (Figure 5(b)) has an imaginary part (dotted line) as well as a real part (solid line), which means that the control source is driven at a different phase from that of the primary source. From

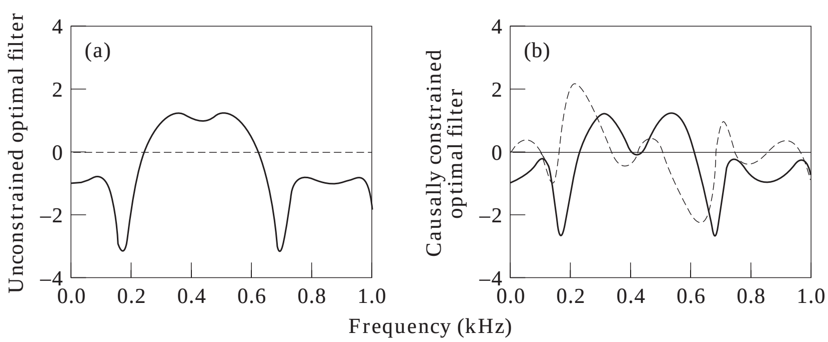

Figure 5. The frequency characteristics of the unconstrained optimal filter (a) and the causally constrained optimal filter (b) in the lightly damped one-dimensional duct system (Figure 3). ——, real part; - - - -, imaginary part.

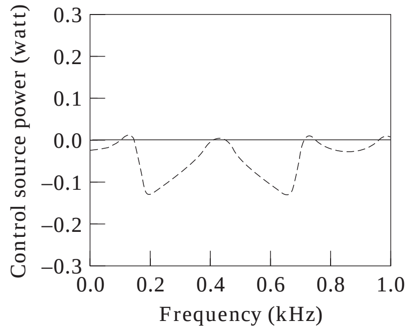

Figure 6. The control source power output on a linear scale after causally constrained optimal filtering in the lightly damped one-dimensional duct system (Figure 3).

the comparison of Figure 5(a) with Figure 5(b), one can also see that the real part of each optimal filter, which represents the components of the control source that are in phase with the primary source, shows a similar trend.

In order to investigate the behavior of the causal optimal control source, the frequency characteristics of control source power were calculated as in Figure 6. One can note that the control source power, which should be exactly zero in the deterministic sound power control situation, as discussed in references [3–5], is not maintained at zero after control. On the contrary, we can see that the control source acts as a sound power absorber for a large portion of the frequency components, as the negative values in Figure 6 imply. This fact in sound power control is analogous to the behaviour of the optimal control source in minimizing the total acoustic potential energy in enclosures [8, 9], even though these two strategies, of minimizing the source power output and minimizing the energy, use different criteria to adjust the control source.

4. CONCLUSIONS

In this study, a causality analysis of the *sound power based active global noise control system in enclosed space* has been accomplished, based on the classical Wiener filtering technique. For simplicity, a two monopole source system surrounded by enclosure boundaries which have arbitrary acoustic impedance condition was considered, which turned out to be a multiple-error Wiener filtering problem which has six errors. A theoretical causal IIR filter and a causal FIR filter for minimizing total sound power in an enclosed space under white random excitation have been derived. For comparison with a causally constrained optimal solution, the unconstrained optimal solution has also been obtained. From the obtained optimal filters, one can estimate the theoretical performance limit of an active sound power control system in an enclosed space. That is, the causal optimal filter provides the lower bound of control performance and the unconstrained optimal filter provides the upper bound.

Numerical simulation for a simple one-dimensional sound field considering plane waves has shown a strong possibility for the reduction of the total acoustic potential energy in an enclosed space using causally constrained optimal filtering which minimizes the total sound power. The simulation result for the random sound field shows that the fundamental controllability determined by the wavelength and location of the control source is still maintained, even if the control performances in the overall frequency range are degraded due to the lack of predictability of the primary random excitation signal. The simulation result has also revealed that a large portion of the control source power shows a negative value, which means that the control source behaves as an active sound power absorber. This result of power absorption is clearly different from the well-known property of an optimal control source used to minimize the total source power output in the deterministic sound field; i.e., in the deterministic sound field, the control source neither radiates nor absorbs any net acoustic power under optimal conditions when minimizing the total sound power of sources.

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APPENDIX: OPTIMAL CAUSAL SOLUTION

One denotes by $y(t)$ the left side of the equation (18a):

$$y(t) = g_{ps}(-t) + g_{sp}(t) + \int_{-\infty}^{\infty} h_0(\tau)[g_{ss}(-(t-\tau)) + g_{ss}(t-\tau)]\,\mathrm{d}\tau. \qquad \text{(A1)}$$

Clearly,

$$y(t) = 0 \quad \text{for} \quad t \geqslant 0, \qquad h_0(t) = 0 \quad \text{for} \quad t < 0. \qquad \text{(A2a, b)}$$

It suffices, therefore, to find a causal function $h_0(t)$ and an anticausal function $y(t)$ satisfying equations (A1) and (A2).

One denotes the $Y(s)$ and $H_0(s)$ the Laplace transforms of $y(t)$ and $h_0(t)$, respectively. As is well-known [13], $Y(s)$ is analytic for $\mathrm{Re}\,[s] < 0$, since it is an anticausal function, and $H_0(s)$ is analytic for $\mathrm{Re}\,[s] > 0$ since it is causal. The integral in equation (A1) means a convolution of $[g_{ss}(-t) + g_{ss}(t)]$ with $h_0(t)$. Taking Laplace transforms of both sides, one concludes, therefore that

$$Y(s) = G_{ps}(-s) + G_{sp}(s) + H_0(s)[G_{ss}(-s) + G_{ss}(s)]. \tag{A3}$$

Our goal is to find two functions, $Y(s)$ and $H_0(s)$, satisfying equation (A3) and the stated analyticity conditions:

$$Y(s) \quad \text{for} \quad \mathrm{Re}\,[s] < 0 \qquad \text{and} \qquad H_0(s) \quad \text{for} \quad \mathrm{Re}\,[s] > 0. \tag{A4}$$

In equation (A3), we let

$$G_{ss}(-s) + G_{ss}(s) = 2R_{ss}(s), \tag{A5}$$

where $R_{ss}(s)$ is the real part or resistive part of the acoustic radiation impedance of the secondary source when it is operating alone; i.e., $R_{ss}(s) = \mathrm{Re}\,[G_{ss}(s)]$. The real part of the radiation impedance of a source determines the acoustic radiation power of the source itself. Therefore it cannot be negative (it would be zero if an acoustic field was ideally reactive; that is, the system was surrounded by ideal rigid walls), and it is even symmetric on the frequency domain. These properties of the real part of the radiation impedance are analogous to those of the autospectrum of a signal. Hence, the factorization of the real part of acoustic radiation impedance is possible using the spectral factorization theorem [11–13]. That is, one can express $R_{ss}(s)$ as

$$R_{ss}(s) = R_{ss}^{+}(s)R_{ss}^{-}(s), \tag{A6}$$

where the $R_{ss}^{+}(s)$ represents the minimum phase system part, and the remaining $R_{ss}^{-}(s)$ the non-minimum phase part. That is, the function $R_{ss}^{+}(s)$ and its inverse $1/R_{ss}^{+}(s)$ are analytic for $\mathrm{Re}\,[s] > 0$, and the function $R_{ss}^{-}(s)$ and its inverse $1/R_{ss}^{-}(s)$ are analytic for $\mathrm{Re}[s] < 0$. To factorize $R_{ss}(s)$, one assigns the left-hand plane poles and zeros to $R_{ss}^{+}(s)$, and the right-hand plane poles and zeros to $R_{ss}^{-}(s)$.

Substitution of equations (A5) and (A6) into equation (A3) gives

$$Y(s) = G_{ps}(-s) + G_{sp}(s) + 2R_{ss}^{+}(s)R_{ss}^{-}(s)H_0(s),$$

$$\frac{Y(s)}{R_{ss}^{-}(s)} = \frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)} + 2R_{ss}^{+}(s)H_0(s). \tag{A7}$$

Then, one can decompose the ratio $[G_{ps}(-s) + G_{sp}(s)]/R_{ss}^{-}(s)$ into causal and anticausal parts:

$$\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)} = \left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{+} + \left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{-}, \tag{A8}$$

where the subscript "+" sign denotes the causal part that is analytic for $\mathrm{Re}\,[s] > 0$, and the subscript "−" sign denotes the anticausal part that is analytic for $\mathrm{Re}\,[s] < 0$.

If the ratio is rational in s, to decompose it into the causal and anticausal parts, one can expand it into a sum of linear terms (partial fraction expansion). One assigns the terms the poles of which are on the left-hand plane to the causal part, and the remaining terms to the anticausal part.

Substitution of equation (A8) into equation (A7) then yields

$$\frac{Y(s)}{R_{ss}^{-}(s)} = \left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{+} + \left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{-} + 2R_{ss}^{+}(s)H_0(s). \tag{A9}$$

Since $H_0(s)$ is causally constrained, the unknown causal optimal filter $H_0(s)$ can be obtained from the equality for the region $\mathrm{Re}\,[s] > 0$;

$$0 = \left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{+} + 2R_{ss}^{+}(s)H_0(s).$$

Therefore,

$$H_0(s) = -\left[\frac{1}{2R_{ss}^{+}(s)}\right]\left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{+}. \tag{A10}$$

Clearly, $H_0(s)$ is causal, because the functions $1/R_{ss}^{+}(s)$ and $[\{G_{ps}(-s) + G_{sp}(s)\}/R_{ss}^{-}(s)]_{+}$ are, by construction, analytic for $\mathrm{Re}\,[s] > 0$. To complete the solution procedure, one can obtain the auxiliary result from the consideration for the region $\mathrm{Re}\,[s] < 0$;

$$Y(s) = R_{ss}^{-}(s)\left[\frac{G_{ps}(-s) + G_{sp}(s)}{R_{ss}^{-}(s)}\right]_{-}. \tag{A11}$$

One can readily see that $Y(s)$ is anticausal by construction.

Finally, one can obtain the frequency response characteristics of the optimal causal filter from the relationship between the Fourier transform of a causal function and the unilateral Laplace transform [13]:

$$H_0(\omega) = -\left[\frac{1}{2R_{ss}^{+}(s)(\mathrm{j}\omega)}\right]\left[\frac{G_{ps}(-\mathrm{j}\omega) + G_{sp}(\mathrm{j}\omega)}{R_{ss}^{-}(\mathrm{j}\omega)}\right]_{+}. \tag{A12}$$